ON THE TREATMENT OF THE ELEMENT OF TIME IN ECONOMICS

By

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I — INTRODUCTION

1. *Time is a centre of difficulty in economics* — The main purpose of this paper is to show the kinds of difficulty to which the presence of the time-element (i. e. the fact that all economic phenomena, like all other phenomena of life, have, so to speak a time-dimension) has given rise, and to investigate the various attempts that have been made to obtain an adequate treatment of that element — attempts ranging from the crude method of trying to assume away the problem, to the highly complicated methods involving the use of advanced mathematics.

II — STATIC, STATIONARY AND DYNAMIC

2. *The treatment of time* — As a result of the great difficulties involved in the treatment of time in economics, initial attempts to take account of that element were crude and inadequate. As the need for a more adequate treatment grew, the revolt against the older methods and the proposals made by their opponents became a central domain of controversy which was christened as the controversy between 'statics and dynamics'.

3. *Statics versus dynamics* — The occasion which gave strong impetus to the opposition between statics and dynamics was the rise of business cycle theory. Beginning with Clément Juglar, explanations of cyclical fluctuations by outside causes, i. e. causes outside the system of explanation, were being discarded in favor of 'endogenous' explanations. This threw a great deal of doubt on the usefulness of the traditional static methods. The cleavage between

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these latter and business cycle analysis which tended to be identified with 'dynamics' was also intensified by the cumulative changes in the techniques of investigation. This is why it came to be a major issue to clarify the relation between statics and dynamics.[1]

4. *Statics and dynamics in mechanics* — In theoretical mechanics, dynamics is that part of the theory which is concerned with moving bodies, while statics is that part which treats of equilibrium or rest. The criterion for the distinction is that of 'motion and rest.' Some writers in economics, however, discard this criterion on the basis that motion does not involve only variation through time, but also of space.[2]

It is to be acceded that, although both economics and theoretical mechanics purport to analyse variation through time, it is variation in space which the latter primarily investigates, whereas the variables in which the former is interested are of a different nature (production and employment, consumption and welfare, etc.). 'Variation through time' would in fact express more accurately what is usually meant; although there should be no confusion in employing the expression 'movement through time', since spatial variation is obviously not meant.

5. *Time relationships and dynamics* — Hicks gives the following distinction between statics and dynamics: "I call economic statics," he says, "those parts of economic theory where we do not trouble about dating; economic dynamics those parts where every quantity must be dated".[3] This definition was criticised by Samuelson as being "overly general and insufficiently precise" since "a historically moving static equilibrium would certainly require dating of the variables, but it would not thereby become dynamic".[4] It is obvious, however, that this criticism is directed only to Hicks' formulation of his definition, not to its actual application. What he clearly means is not calender-dating, but caring in general about the time-relationships of the variables concerned.

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1. Simon Kuznets, "Equilirium economics and business cycle theory," *Quarterly Journal of Economics*, vol. 44, pp. 381—415.

2. N. D. Kondratieff, "The static and the dynamic views of economics," *Quarterly Journal of Economics*, vol. 39, pp. 575—583.

3. J. R. Hicks, *Value and Capital*, 2nd edition, p. 115.

4. P. A. Samuelson, *Foundations of Economic Analysis*, 1947, p. 315.

6. *Is statics a special case of dynamics?* — The object of economic dynamics is thus the determination of the variables involved as functions of time, on the basis of our knowledge of the initial values of these variables and their modes of variation. The case where such variables do not change their values with respect to time is a special case of functional variation which economic statics proposes to investigate. Thus economic statics is only a special case of economic dynamics. A society which such a system purports to describe is called 'stationary'.[1]

It should be noted that a static system of explanation may be timeless in spite of the fact that its final objective is to explain the functioning of an economic system moving through time. A distinction should be made between the term characterising the tool of analysis (the static analytical system), and the economic system which is to be explained (the stationary type of society). The intimate relation between these two different concepts arises from the fact that, because the stationary state is characterised by repetition of the same economic processes, the explanation of any instantaneous cross-section of the flow would be at the same time an explanation of the flow itself. This however is no excuse for confusing the analytical device with the object it purports to analyse.

Some writers, however, prefer, for analytical reasons, to so define economic dynamics as to exclude from their definition the statical types of analysis. This is what Samuelson, e. g., follows. He states three alternative ways of defining economic dynamics: (1) The above mentioned definition of dynamics as the analysis intended to determine the variables concerned as functions of time, where economic statics is considered as a degenerate special case. (2) The same definition only with adequate allowance for heavy dampening of the economic system analysed of which the static case would be a limiting case in which the dampening ensures instantaneous adjustment. Statics here is also a special case of dynamics. (3) The definition which is chosen by Samuelson is that "a system is dynamical if its behavior over time is determined by functional equations in which variables at different points in time are involved

1. E. Lindahl, *Studies in Theory of Money and Capital*, p. 31.

in an essential way". Such a definition excludes the statical types.[1] It is to be noted that, for this definition, time should be involved in an essential way. Statical systems do not become dynamical by introducing into them dynamical refinements.

7. *Refined statical systems* — The dichotomy between statical and dynamical is primarily schematic since very few economists were ever completely statical. A typical statical system may be offered from time to time by writers who profess to use it only as an introduction to the more complicated dynamic analysis; this is the case we have, e. g., in Hicks' *Value and Capital*, who hastens, however to remind us that "indeed it was only because I had a dynamic theory in preparation that I could dare to make my static theory so static." [2]

Although statical analysis always starts from a timeless phase refinements are usually introduced by taking into account rates of change, anticipations, durability, the terms of a loan, and similar refinements. And yet, these refinements do not afford enough accounting for the various aspects of time-differences. In particular, as E. Lundberg points out,[3] so long as the variables taken into account are considered simultaneously variable, the analysis is still static.

8. *Comparative statics* — From its analysis of an instantaneous cross-section of an economic system, the statical method can tell us only whether the economic system at the particular point in time is or is not in equilibrium. It can indicate to us the conditions that would be necessary for equilibrium under given initial conditions. It can thus give us an idea about the various equilibria that are to be expected from variation in these initial conditions. This latter function is what is termed 'comparative statics'. This should make it clear that it is not the object of comparative statics to 'describe the transitional path between equilibria'.

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1. P. A. Samuelson, *ibid.*, pp. 284—285 and p. 314.

2. J. R. Hicks, *ibid.*, p. 115.

3. E. Lundberg, *Studies in the Theory of Economic Expansion*, ch. 1, section 2.

The neglect, however, of the investigation of the transition between equilibria may lead to serious error. An illustration of this is afforded by the curious discussion that has taken place between Pigou, on the one hand, and Keynes and his followers, on the other, about whether a money wage-cut affects employment through, or independently of its effect on the interest-rate.[1] As Lerner pointed out, the root of the difficulty in the controversy was the insistence of Pigou on analysing the problem in terms of comparative statics, i. e. concentrating his attention on the equilibrium positions, without investigating the path by which that equilibrium is attained. [2]

As we shall point out later, the belief in the usefulness of comparative statics depends essentially on the belief in the stability of the economic system, i. e. the belief that the system is sufficiently damped to ensure convergent behavior. If this is the case, an adequate notion as to the actual path to equilibrium can be derived from investigating the equilibrium situations themselves. [3]

III — TYPES OF EQUILIBRIUM ANALYSIS

A. Partial Equlibrium Analysis

9. *The Marshallian system* — The classical example of partial analysis is undoubtedly the Marshallian system as put forth in his *Principles of Economics.* The Marshallian system is concerned primarily with the phenomena of the market. In this tradition, the economic unit is the industry, i. e. a group of firms producing an undifferentiated product; the single firm being treated only in the case of monopoly, i. e. where the firm is identical with the industry, and also detachedly, a propos the problem of increasing returns, industry being exemplified by the 'representative firm'.

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1. A. C. Pigou, "Real and money wages in relation to employment," *Economic Journal* vol. 47, 1937, pp. 405—422. He there maintains that the effect of a money-wage cut is to increase employment independently of its effect on the interest rate.

2. A. P. Lerner, "Ex-ante analysis and wage theory," *Economica*, N. S., vol. 6, 1939, pp. 436 — 449. See also the discussion between N. Kaldor and H. M. Somers about the confusion resulting from applying two theories of the interest rate, *Review of Economic Studies* vol. 5—8.

3. Samuelson, *ibid.*, p. 331.

The type of interdependence analysed in the Marshallian tradition is the interdependence within the industry, so-called 'internal interdependence', in contrast to 'external interdependence' where the relation among the various economic units is analysed.[1]

10. *Marshall's protest against characterising his method as static: complexity of the biological concept* — The steps in the analysis under-taken by the traditional method can be summarised as follows: In the first place, the analysis enumerates the factors that affect the formation of price: tastes, productive capacities, the distribution of population and of wealth, the state of the technology, the state of elaboration of exchange, and so forth. Secondly, it holds all these factors constant and derives on the basis of that assumption the supply and demand curves. Thirdly, on an implicit belief in stability, the system is supposed to move towards the attainment of equilibrium through a process of trial and error, whether the equilibrium position is stated to be one in which marginal cost and price are equated (when perfect competition is assumed), or one in which marginal cost and marginal revenue are equated (when imperfect competition is accounted for). The fourth and last step is a concession to the effect that the position of equilibrium is only the centre to which the system at any moment gravitates; the analysis indicates a tendency to the equilibrium position, not that this position is actually realised.

The frequent use by this approach of the concepts of normality and equilibrium and of the various forms of mechanical analogy, has laid it open to the common criticism that it is statical. Marshall, however, does not yield to such criticism. "In fact it is concerned throughout with the forces that cause movement", he asserts, "and its keynote is dynamics rather than statics".[2] The necessity for simplifying devices is explained by the complexity of the forces dealt with. Where the investigator is confronted with such complexity, the rational way to proceed into the analysis is to

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1. R. Triffin, *Monopolistic Competition and General Equilibrium Theory*, pp. 3—15.

2. Marshall, *ibid.*, pp. xiv-xv.

take few forces at a time. Then, as the influence of these becomes more or less acquainted with, "more forces are released from their hypothetical slumber".[1]

11. *The stationary state as a first step in partial analysis* — The fact that partial analysis intends the fiction of stationary conditions as a first step in the analysis is explicitly stated by Marshall. He defines the stationary state as that state in which the "general conditions of production and consumption, of distribution and exchange, remain motionless". It is to be noted that what is defined as remaining motionless is not the economic system, since the economic system is never motionless; what are assumed to be motionless are the 'conditions' underlying supply and demand. This stationary state is not investigated, however, for its own aske; for, per se, it is insignificant and irrelevant to the conditions of actual life. It is a means of making applicable the unsophisticated statical method.[2] For the great advantage of the assumption of stationary conditions lies in its implication of the constancy of the stock of capital, i. e. in the absence of net capital accumulation (investment); since then, according to the famous illustration given by Pigou, the existing capital stock can be looked upon as a lake, the level of which is kept constant as a result of a constant equal inflow and outflow of water, both of which are by no means simultaneous but can be treated as such. In general, therefore, this is a method to abstract from the time occupied by the process of production; inputs and outputs can be treated as simultaneous.[3]

When this simple state is investigated, the next step is to relax the drastic assumptions made so as to get nearer to a more realistic picture of life.

12. *The tripartite division of adjustment-time* — The shocks coming to a stationary system from the relaxation of stationary conditions throw that system into disequilibrium accompanied by a

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1. There is, however, a curious problem of whether, because of their complexity, biological phenomena need a fundamentally different method of investigation from that which is applied in physics and mechanics. The answer should be in the negative since complexity does not mean the denial of the applicability of tools useful in analysing the more simple cases. See Samuelson, *ibid.*, p. 312, n. 6.

2. Marshall, *ibid.*, pp. 366-367.

3. Hicks, *ibid.*, pp. 117-118.

tendency to adjust to the equilibrium position defined by the new conditions. The traditional analysis, however, takes account of the fact that adjustment takes time and that the rate of adjustment differs as among the different factors involved, by examining equilibrium after different spans of time. This is the method adopted by Marshall in his tripartite division of adjustment time. "We shall find", says Marshall, "that if the period is short, the supply is limited to the stores which happen to be at hand; if the period is longer, the supply will be influenced, more or less, by the cost of producing the commodity in question; and if the period is very long, this cost will in its turn be influenced, more or less, by the cost of producing the labor and the material things required for producing the commodity".[1] That the implication of this procedure is the attainment of a new stationary state is clear from the concept of the long run, provided no new disturbances take place. For, as Marshall himself notes, the theoretically perfect long period requires the adjustment of the factors of production and the factors necessary to produce these factors, and so on, which is another way of saying that it implies the stationary conditions.[2]

B. General Equilibrium Analysis

13. *A different theory of the economic unit and of interdependence* — General equilibrium analysis is subjective: its economic unit is the 'individual', whether a household or a firm. This distinguishes it from the Marshallian type of analysis where the individual lies in the background. Another distinguishing feature is that, instead of focusing on the conditions of 'internal equilibrium', i. e. equilibrium within the economic unit, it pays much attention also to the problem of 'external interdependence', i. e. interdependence among the different units.[3]

14. *The Walrasian general equilibrium system is static* — The problem which Walras investigated was that of finding the solution for a set of simultaneous equations descriptive of certain initial conditions. His broblem was that of the 'determinateness of

1. Marshall, *ibid.*, p. 330.
2. Marshall, *ibid.*, p. 379n.
3. Triffin, *ibid.*, pp. 3—15.

equilibrium' on the statical level. This was the stage which Samuelson refers to as the stage of counting equations and unknowns. The system is therefore static; the investigation of the uniqueness of the stability of the solution is a mere corollary of the main problem. The process of trial and error (tâtonnement) does not change the essentially static character of the analysis.

We shall come later to the discussion of whether it is more desirable to build a dynamic system on the basis of the already known static system. It suffices to point out now that attempts to introduce time into the Walrasian system have been laborious and not always very successful.[1] Many writers therefore advocate the abandonment of the statical approach in favor of a more promising one.

C. Aggregative Equilibrium Analysis

15. *A new approach?* — Total or aggregative analysis is concerned with total categories, i. e. the summation of individual categories. Hence, partial analysis is in this sense also aggregative, although it is concerned only with aggregates for an industry. Total analysis applies more specifically to those types of analysis concerned with aggregates for the whole economy.

16. *Type of aggregative analysis* — Different models of aggregative analysis are arrived at according to the various assumptions that can be made with regard to the variability of the factors involved. A given system of explanation considers a factor variable within the system if account is given of its mode of variability on the basis of the given initial conditions. Other variables may be considered as varying outside the system of explanation in the sense that the system does not consider itself responsible for affording an explanation of their variation. Obviously, the explanatory value of any system must depend on its 'degree of variance', i. e. on whether its choice of what is variable and what is not conforms to real life.[2]

The existence of money, and the resulting influence of the variation in its quantity and its price, have been abstracted from in a number of aggregative analyses, the so-called real analyses.

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1. Lindahl, *ibid.*, p. 33.

2. Lundberg, *ibid.*, ch. II, sect. 3.

Of this type is, e. g., Hayek's model in *Das Neutrale Geld*, where he abstracts from the influence of money by assuming away the existence of savings. Robertson also, in *Banking Policy and the Price Level* makes the same abstraction by expounding his system in real terms through the transfer of the partial method to total analysis. It is obvious that such methods have the advantage of bringing out the influence of real factors on the economic system, but it hardly needs mentioning that this is only part of the explanation of real life.

Another factor, the assumptions with regard to which have given rise to various models is 'capital'. Examples of systems based on the assumption of the variability of the stock of capital are afforded by the Böhm-Bawerkian and the Wicksellian systems. The core of these systems is the phenomenon of the variation of the stock of capital through the variation in the degree of rounaboutness of production (i. e. in the marginal productivity of time).[1] Other systems take the quantity of capital as given. Of this type is the Keynesian system. This latter system is, par excellence, the commonly applied type of aggregative analysis.

17. *The Keynesian system* — The Keynesian system has been so frequently analysed by different writers, that we shall confine ourselves only to a cursory survey of it. Beginning with given supplies of labor and capital, Keynes builds his system on three psychological reaction functions : (a) The consumption function; (b) The schedule of liquidity preference; and (c) The schedule of the marginal efficiency of capital. With a given quantity of money, the system is determined: 1) From the schedule of liquidity preference, the quantity of money determines the interest rate. 2) The interest rate determines the amount of investment from the given marginal efficiency of capital schedule. 3) From the amount of investment, the total amount of income is determind through the investment multiplier which can be derived from the consumption function. Only expectations of these categories which are mutually consistent are realised. This analytical framework is used by Keynes to show that however large the quantity of money is, the rate of interest does not fall sufficienly to ensure full-employment, since after a certatin limit it stops falling because of the expectation of its rise. This is taken as a basis for justifying supplementing private by government investment.

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1. Lundberg, *ibid.*, ch. 2, sect., 4.

18. *Is the Keynesian system static or dynamic ?* — From this quick survey of the Keynesian system, it is obvious that it can only tell us what conditions are necessary for the attainment of full-employment, and why the economic system can be in equilibrium before that stage has been attained. It does not analyse the process of change of the variables involved in the economic process. The Keynesian system is then of the nature of the traditional 'comparative static' type. It is obviously a refined type of comparative statics, in which a great deal of dynamic elements are involved. The most important among these should certainly be the analysis and incorporation into the system of the factor of 'expectations'. An attempt to construct a more general dynamic model which includes the Keynesian model as a special case has been undertaken by Samuelson.[1] In fact he proposes two different models: (*a*) A model based on a differential equation which makes the rate of change in income with respect to time a function of the difference between intended saving-investment and actual saving-investment[2]; and (*b*) A model based on a difference equation in which consumption in one period is considered as being partially a function of the income of the preceding period. Samuelson uses these two models to determine the possible signs of variability of the involved unkowns with respect to the given parametric functions. The method in fact helps to clarify some possible modes of variation; although the final conclusion arrived at by Smuelson is that "the only theorem which remains true under all circumstances is that an increase in the amount of money must lower interest rates if the equilibrium is stable".

IV — THE LIMITATIONS OF EQUILIBRIUM ANALYSIS

19. *Marshall's own conception of these limitations : the analysis of progress and development* — We have had a chance to point out above Marshall's concern about treating what are in fact biological phenomena with the tools of a mechanical method. Marshall's defense of such a procedure was, we remember, that it is a

1. Samuelson, *ibid.*, pp. 278-283.

2. What Samuelson means is to assume the mutual consistency of saving and investment plans. Thus saving-investment whether planned or realised would be equal. Intended saving-investment in this sense however can be disappointed through inconsistency with other factors.

necessary introduction, made so by the extreme complexity of the phenomena studied. For this reason, he warns against expecting such a method to be helpful in analysing the long-run problems of progress and development. For such analysis, not many factors can be assumed constant. The method breaks down, and we are more or less constrained to have recourse to "a more philosophical treatment of society as an organism." We have pointed however to the mistake of believing that, because of their complexity, biological phenomena need any fundamentally different method of attack than that applied to physical phenomena. It is not to be doubted that his assertion as to the crudeness of the statical method is correct, but the remedy should lie in the use of the more advanced equipment of handling physical phenomena.

20. *The case of increasing returns* — The same general concept of the biological nature of economic phenomena is applied to the business firm. The temptation of applying to it the tool of static equilibrium leads to the neglect of the fact that it is always in a state of organic growth or decline, through extending its internal and external economies in the one, and through the rise of competitors or the decline in its managerial effectiveness in the other. In the short run, the adjustment of the firm to internal and external economies is confronted with difficulties so great that it is more likely to be subject to the law of increasing costs. In the long run, however, more time is allowed the firm to adjust, so that the phenomenon of increasing returns may appear. It is thus because of the biological nature of the firm, because of its constant evolution, because it is never in the equilibrium in which the economist puts it, at least in the long run, that it may in fact be found producing subject to increasing returns. Marshall therefore blames the statical equilibrium method for its impotence in explaining this case of in creasing returns. [1]

This final judgement rendered by Marshall does not however mean that he was unaware of other possible explanations of the difficulties experienced in the analysis of the case of increasing returns. In fact, he offers the explanation that has later become popular through the writing of monopolistic competition theorists. "It should however be noted that in many industries each producer

1. Marshall, *ibid.*, pp. 500—501.

has a special market in which he is well-known, and which he cannot extend quickly; and that therefore, though it might be physically possible for him to increase his output rapidly, he would run the risk of forcing down very much the demand price in his special market, or else of being driven to sell his surplus production outside on less favorable terms."[1] This is in other words nothing more nor less than the familiar monopolistic competition explanation based on product differentiation as put forth by Chamberlin. Yet still Marshall does not consider this the basic explanation. His basic explanation is that industries which are subject to increasing returns are "in a transitional state, and it must be conceded that the statical theory of equilibrium of normal demand and supply cannot be profitably applied to them".

21. *The recognition of continuous change and of time-discrepancies* — After having considered 'the famous fiction of the stationary state', Marshall plainly recognises the limitations of such an approach. In the first place, the assumption of constancy of certain factors is far from reality: "Every economic force is constantly changing in action, under the influence of other forces which are acting around it". In the second place, although the primary utility of the fiction of the stationary state is to enable us to get around the time factor involved in the production process, no attempt is made to conceal the real nature of the categories involved. It is explicitly stated that "all these mutual influences take time to work themselves out." In the third place it is frankly stated that "as a rule, no two influences move at equal pace." The factor of time-discrepancy between different effects is well stated. It should be noticed however that no attempt has been made on the part of Marshall to construct an adequately dynamic system to take account of these elements. Obviously, what he had in mind is to give an approximation to the facts through his avowedly inadequate analysis and to provide at the same time for the necessary qualitative reservations by pointing out the limitions of his method[2].

22. *The revolt against statics : the attack on the ceteris paribus device* — In the light of the pressing need for a dynamic system, the shortcomings of statics became obvious, and economists

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1. *Ibid.*, p. 501.

2. Marshall, *ibid.*, p. 268.

embarked on a severe onslaught on that method. The need for dynamics arose from two facts. First, the fact that real phenomena are always changing and that mere qualitative apprehension of their change was deemed insufficient; and, second, the tremendous importance which the phenomenon of economic development has come to assume at the hands of modern economists.

This onslaught on statics took various forms each of which concentrated on a particular aspect of that method. One line of attack was that directed against the *ceteris paribus* device which is basic for the static method.

Writers who attacked this device differentiate between the use made of it in mechanics and that in economics. In mechanics, they maintain, this device is acceptable since the condition of the independence of forces is satisfied. This, however, is not the rule in economics since factors interact so that a change in market conditions usually does not leave other things equal. In the first place, such a change in market conditions changes other factors quantitatively, as in the case of the effect of a change in wages on efficiency. In the second place, it changes them qualitatively, as in the case of the effect of a change in knowledge on the quality of competition (through combinations, etc.). In the first case, we have quantitative interdependence among factors; in the second, we have qualitative interdependence. The existence of this latter type of interdependence is especially important since it does not only mean that static assumptions have to be modified quantitatively to conform to reality, but also that they are qualitatively invalid.[1]

The real importance of this criticism lies in the warning it gives against neglecting the influnce of the economic process on the basic initial conditions which the analysis assumes. It is not, however, a serious criticism if we grant static analysis the common argument that it is merely a step towards a more complicated approach. Whether we do in fact grant this argument is a matter of personal opinion, but the fact remains that we need the more complicated approach, and this criticism has the advantage of making us bear in mind one of the elements that are abstracted from.

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1. M. Abramovitz, *Price Theory for a Changing Economy*. See also J. M. Clark, "The relation between statics and dynamics", in *Economic Essays in Honor of J. B. Clark.*

23. *The impotence of static analysis in analysing development: the escape throught the concep of 'exogenous factors'* — The assumption of 'ceteris paribus', together with the belief in the convergence of the system towards equilibrium, end ultimately in the necessity of a stationary state in the long run : a fiction which is so far from reality that some explanation of actual development, even within the framework of the industry which partial equilibrium analysis takes as its umit, has to be given. The static analytical system itself cannot give such an explanation; hence it is forcèd to ascribe development to factors outside the scope of that system. These are the so-called 'exogenous factors'. It is obvious that, if these factors are to be incluḍed in the system as variables and not as data, the ceteris paribus device has to be abandoned. This device, however, is the backbone of partial analysis; hence the conclusion that if we want to have an explanation of development which is at the same time part of our general system of explanation, partial analysis has to be discarded[1].

24. *The sum of partial analyses does not explain 'total' development: the error of transfer* — Even if we accept the above method of partial analysis as a moderately adequate way of providing an idea of development in the particular industries, this method is completely incapable of explaining total development. The reason for this is once again the 'ceteris paribus' assumption. For if it is acceptable to take other things as constant when one is speaking of a single industry, there is much less ground for making such an assumption when the total economy is the subject of investigation. The change in the categories assumed constant is in many cases the precise objective of total analysis, as is the case with total income.

The method of explaining the total economic process with the aid of partial explanations has resulted in what is commonly termed 'the error of transfer', i. e. the error of transferring partial analysis to the analysis of the whole economic system. Here, the picture of the economic system is given as a collection of partial markets in which the relative prices of the different commodities are determined, with the quantity of money coming in to determine the actual price-level. When such a procedure, is followed the limitations of the partial

1. Lundberg, *ibid.*, chapter 1, sections 1 & 5.

method arising from the 'ceteris paribus' clause are neglected. This is the source of the traditional cleavage between 'real' and 'monetary' economics.

25. *In the long-run, things do not remain constant* — Even within the realm of the individual industry, partial analysis is confronted with insuperable difficulties when it comes to the discussion of long run effects. The reason for this is that effects are analysed on the basis of the given initial conditions which are assumed unchanged. In the short run, these initial conditions can be assumed unchanged as a form of approximation to reality, since their rate of change throught time is relatively small; in the long run such an assumption is unjustifiable. The result is that partial analysis is more of an approximation to reality when it deals with short run effects than when it deals with the long run.

26. *The revolt against statics: the attack on the tendency towards equilibrium* — We have already mentioned the fact that the greatest impetus to dynamic analysis came from the attempt on the part of the economist to deal with the phenomena of the business cycle. In fact, in the face of persistent cyclical fluctuations, economists were justified in their disappointment with a body of theory that affirmed the existence of a strong tendency towards equilibrium and that movement away from equilibrium is but a temporary aberration. In the search for a more adequate explanation of reality, economists did their best to find out the reasons for the failure of the equilibrium method. Various views were offered.

Some writers (e. g. E.Lederer) criticised equilibrium theory for not taking into account the type of 'social economy' in which we live. By social economy, he obviously means the existing economy with all its sociological elements, as distinct from a hypothetical 'pure' economy. This in itself is sufficient to indicate to us the limitations of such criticism. For, in the first place, pure economics in a strict sense cannot possibly exist. In the second place, the inclusion in a system of explanations of sociological characteristics of an economy is a matter of judgement based on the needs of the investigation at hand. Still the criticism is quite significant in the sense that traditional economic theory excluded from its framework sociological characteristics which are deemed to have important explanatory value.

Other writers attack the validity of the traditional assumptions underlying equilibrium analysis. Of this group is A. Loewe. He analyses the different business cycle theories and notices that most of them attack some of the basic assumptions of equilibrium analysis. The anti-theoretical group of business cycle theories (e. g. theories of Hardy and Pigou) attack the assumption of rationality. Others like Aftalion and Cassel take for granted the existence of unstable conditions and start their analysis by assuming a phase of the business cycle. The quantity theorists, along with Sombart and Liefmann attack the assumed cancelling influence of the system; partial disturbances are presented as giving rise to general ones. Fisher stresses the phenomenon of time-discrepancies. Thus, a large number of business cycle theories try to find an explanation for the business cycle in denying an assumption made by equilibrium analysis.

This criticism of equilibrium theory is not well-received by many writers. They deny that equilibrium economics assumes without qualification complete rationality, immediate adjustment or equal time spans of adjustment. This is obviously true, but it neglects the fact that all this is introduced into the system merely as qualifications not as an integral part of its structure and reasonning.

In defense of equilibrium analysis, writers like Carrel define economic theory as the necessary relationships derived from the two basic assumptions of scarcity and rational action, thus implying that economic theory is not required to conform to facts or to explain them, since it is merely a logical structure based on given initial assumptions. All that should be required of economic theory according to this view is logical consistency.

This, however, is an extreme conception of what the role of economic theory is. Economic theory is not supposed to be merely a collection of mental exercises. Society is confronted with problems which it is essential for it to solve; and the role of all speculation should be to do its share in helping to solve these problems. This requires that economic analysis conforms as much as is feasible to actuality to the degree that is required by the type and broadness of the problem. It is a valid crticism therefore to contend that the basic assumptions of a given analysis are remote from the facts which it purports to explain, although in many

cases it is true that altermative assumptions render the analysis so complex that enough justification is thereby claimed for the cruder assumptions.

27. *The attack on the tendency towards equilibrium : the system has no tendency to cancell random changes* — The belief in the tendency of the economic system towards equilibrium is based on an implicit belief in the capacity of that system to withstand random changes. Obviously one way in which this belief can be arrived at is by assuming a strong dampening effect in the system. Frequently, however, the cancelling effect is ascribed to a cruder source, namely, that as a result of randomness and proportionality of effects, these latter tend to cancell each other. This false contention has been challenged by many writers. Kuznets, in the abovementioned article, states as one of its main objectives the establishment of the thesis that "the economic system is not a stable system which reacts to random changes by cancelling them instantaneously or after a while". He indicates that taking account of time differences (see below), the possibility of this cancelling of random effects is almost completely excluded, and we have in its place cumulative effects. This is because, in order for random effects to cancell out, they should be more or less proportional, as rates of change per unit of time, and of differently distributed signs. The first of these conditions is not realised because differences in time spans of adjustment mean disproportionality. The second is not realised because, as was shown by the work of the Russian statistician Slutzky, it is in the nature of random frequency that such events would come, from time to time, in clusters of the same sign — a phenomenon which has been used to explain the regularity of cyclical fluctuations by some business cycle theorists.

28. *The exchange process itself affects the final equilibrium : the equilibrium position defined by equilibrium analysis is faulty* — The equilibrium position defined by equilibrium analysis is a deduction from the basic initial conditions which this analysis assumes. No account is taken of the fact that in real life the path by which this final equilibium is attained may affect the position of final equilibrium. This is equivalent to assuming that this final equilibrium position is attained instantaneously, an assertion which would be true if the conditions of perfect competition are assumed. Another way to get around the difficulty arising from the effect of the exchange

process on the final equilibrium has been resorted to by F. Y. Edgeworth; this is to assume that the market permits of recontract. In each of these cases, the final position of equilibrium would be identical with that defined by equilibrium analysis.

Such assumptions are, however, an obvious violation of the conditions of actual life. Perfect competition is far from being the prevalent form of market organisation. The process of recontract is only realised in exceptional markets, such as auction markets. To take these as a typical case for analysis would be merely an escape from the complications of the problem. As Chamberlin puts it, "movements towards and fluctuations about equilibrium characteristically leave a trail of actual prices behind them which may not be revised but which are final."[1]

The result of price fluctuations is that the amount sold will be greater than the amount defined by equilibrium analysis. Put in other words, the result of price fluctuations is that the equilibrium amount as defined by equilibrium analysis should sell, not at the price determined by that analysis, but as a higher price. The reason for this is that "for all prices higher than the equilibrium one, supposedly excluded sellers have a chance to dispose of their goods, and there is no reason why some of them should not do so. Similarly, supposedly excluded buyers may be included when fluctuations carry the price below equilibrium."[2] Thus, the actual volume of sales will lie somewhere between a minimum (the equilibrium amount as defined by equilibrium analysis) and a maximum (the quantity if all excluded buyers and sellers have actually contracted). They obviouly will do that with sellers and buyers who are not excluded; but the result is, from the point of view of the quantity, as though the excluded buyers and sellers have contracted with each other. The quantity bought can be larger than the traditional equilibrium amount by as much as they would contract.

On the basis of this analysis, Chamberlin condemns speculation, since its ultimate result is to increase the price. We shall see below that other writers praise speculation as the force that acts as a coordinator under a non-planned system. We shall leave this discussion until later.

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1. E. H. Chamberlin, *The Theory of Monopolistic Competition*, 1946, pp. 25-9.
2. Chamberlin. *ibid.* p. 27.

29. *Marshall's defense of the procedure of neglecting the effects of the exchange process: the effects are only income effects* — Marshall was obviously conscious of the fact that a difference does exist between a system where adjustment to equilibrium is almost instantaneous as a result of perfect competition and one in which the attainment of equilibrium is a prolonged process of trial and error. Yet, somehow, he did not seem to recognise the possibility that buyers and sellers supposedly excluded according to his analysis would enter the market. On the hypothesis that contracting is still confined to the buyers and sellers defined by equilibrium analysis, he rightly affirms that the effect of the adjustment process is only an income effect. This is because, if the same contactors are in the market, the fluctuation in price is equivalent to an impoverishment of those who buy at higher than the equilibrum price or sell at a lower price compensated by a corresponding enrichment of those who happen to be their partners. These are obviously income effects, in the nature of a redistribution of income between buyers and sellers. And since the part of an individual's income spent on one commodity is usually small, these income effects can be neglected. This is completely in harmony with Marshall's convention of neglecting income effects.

Hicks, however, in spite of the great care that he took in his static exposition to isolate the income effect, agrees here with Marshal that it is probably insignificant.[1] He adds the argument that the fact that gains to buyers mean losses to sellers and vice versa, has a dampening effect on the possible disturbance arising therefrom.

It is however clear that both Marshall and Hicks neglect the possibility analysed by Chamberlin. As he rightly affirms, in the course of price fluctuation both some excluded buyers and some excluded sellers have a chance to contract; and there is no reason at all to suppose that they do not take that chance.

30. *The attack on the tendency towards equilibrium: the possibility of persistent disequilibrium from time-discrepancies* — Among the limitations on the device of describing the economic process as tending towards equilibrium is the possibility that, although the

1. *Value and Capital*, p. 129.

equilibrium position as defined by the traditional analysis does exist, it will not necessarily be reached because of the instability arising from neglected time-discrepancies. The analysis that has been undertaken to demonsrate this possibility is the familiar 'cobweb theorem'.

The name 'cobweb' was given to this theorem by N. Kaldor in 1934. The theorem itself was simultaneously and incidentally developed in its three types by Henry Schultz and J. Tinbergen, both of whom show the importance of a lag in the production response to price changes; and by Umberto Ricci who showed the importance of differences in the numerical values of the elasticities of supply and demand.[1]

When the process of adjustment is examined, interesting possibilities appear. The first attempt towards adjustment must necessarily be insufficient since the classical assumption of perfect knowledge or perfect mobility is not realised. The pattern of subsequent adjustments, however, depends on a number of factors. With regard to these factors, the cobweb analysis makes the following assumptions :

1. It assumes that adjustment is completely discontinuous: each adjustment being achieved in one step.

2. The time required for adjustment is assumed to be identical regardless of the size of the adjustment.

3. No lag exists between changes in the rate of input and changes in the rate of output.

4. The reaction of demand to changes in the market conditions is assumed to be instantaneous.

5. Producers are assumed always to expect the present price to continue.

6. Business carryover is neglected.

According to these assumptions, subsequent adjustment would depend on the numerical values of the elasticity of demand and supply. The continuous type of cobweb is obtained when these

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1. M. Ezekiel, "The cobweb theorem," *Quarterly Journal of Ecnomics*, vol. 52, pp. 255—480. Also in *Readings in Busines Cycle Theory*, pp. 422—442.

elasticities are equal; the convergent type, when supply is less elastic than demand; and the divergent type, when supply is more elastic than demand. The figures for the illustration of these models are familiar, and I shall therefore dispense with them. Leontieff has shown that the supply and demand curves may be of an erratic shape so that they present one type of fluctuation or the other only within limits.

31. *The result of refining the cobweb assumptions* — The assumptions underlying cobweb analysis are to some extent unrealistic. It is interesting to see whether the refinement of these assumptions yields a more or less stable system. In the first place, adjustment is never completely discontinuous except in a number of cases primarily of agricultural commodities. If we assume continuous adjustment of supply the criterion for a convergent system changes. As Kaldor pointed out this criterion becomes whether the rate of adjustment on the supply side is less than that on the demand side. Kaldor's procedure was to devide the period of adjustment of supply into ultra-short subperiods whose corresponding supply curves were derived from the long run supply curve. The same was applied to the demand side. And the technique of cobweb analysis was applied to these ultra-short periods. Kaldor's conclusion, however, is doubted by Abramovitz on the ground that the short run supply and demand curves become so complicated as the periods progress that no definite answer is possible. He remarks, nevertheless, that, since supply becomes more and more inelastic as divergent fluctuation proceeds, there is a limit to divergent movement.

The type of adjustment pattern that is implied in Marshallian analysis is more in conformity with that proposed by Lange. Assuming full and immediate adjustment of demand, it is natural that, if the quantity supplied is not identical with the equilibrium amount, adjustment would come gradually through small steps of increase in supply, price changing continuously until equilibrium is reached.[1]

The possibility that demand may not adjust itself instantaneously is another possible source of instability. The more ready adjustment of demand is certainly helped by the continuity of

1. See Abramovitz, *op. cit.*

adjustment on the supply side, since sudden placement of goods on the market is not likely to meet with an easily adjustable demand. However, even then, demand takes time to get adjusted to new market conditions.

A fertile source of instability is the possibility of erroneous expectations. Erroneous expectations lead to fluctuation which in its turn aggravates the problems of forecasting and renders the making of correct estimates more difficult. The assumption that producers always expect current market prices to continue is not realistic. This effect of expectations becomes more pronounced when we remove the assumption of the absence of any production lag and that of the absence of carry-over. We shall see below, however, that it is usually maintained that it is not the expectation of the equilibrium position which contributes to more stability, but the inelasticity of expectations. The idea is, as we shall find later, that inelastic expectations lead to beneficial effects of substitution over time in a form which will alleviate excess demand or excess supply as the case may be.

32. *Cobweb models with more than a one-period lag* — The common cobweb models are based on a one-period lag between the change in price and the response of supply. If the lag is of two periods, the effect of the price change in the first period will appear, on the assumption of complete discontinuity, only in the third period; the effect of the change in the second period will appear only in the fourth, and so on. We thus obtain models of fluctuation which are similar to the time series of price fluctuation of some of the agricultural commodities, especially cattle and hogs. The same analysis can be applied to more than a two period lag.

33. *The limitation of cobweb analysis* — We have pointed to the fact that cobweb analysis is a good approximation to the price fluctuation pattern of some agriculural commodities. Even in this field, cobweb analysis has limitations. In the first place, the response of the supply of agricultural commodities is usully less on the upward side than it is on the downward side. Secondly, the assumption of a precise lag is not realistic. It is not true, moreover, that the only factor that affects supply is the price of the previous period, although it should be noted that the neglect of other factors is in harmony with the assumption of equilibrium analysis that other things remain equal.

However drastic such limitations might be, the analysis certainly indicates a possible flaw in the structure of equilibrium analysis. The implied attack on the belief in stability was the first step in stability analysis, which, as we shall see below, was an immense impetus to the development of adequate dynamic models. In fact, the cobweb theorem was the first sophisticated model of non-linear dynamic systems that received adequate treatment.[1]

34. *The attack on the tendency towards equilibrium : the influence of expectations* — The infuence of expectations on the stability of the economic system was not investigated to a tolerable extent except comparatively recently. The reason for this is that economists for a long time were in doubt as to whether expectations and the problems connected therewith, e. g. the problem of uncertainty, lie within the scope of economics. In fact, many economists maintained that all such issues were outside the economist's competence (e. g. L. Robbins). If this is accepted, expectations must be taken as data and their variability over time must not be investigated. This attitude towards expectations takes the form of assuming perfect foresight or static expectations, both of which are unsatisfactory as we shall point out later. Evidently, what has to be taken as data in economics and what has to be taken as variable is a matter of the boundaries between the social sciences. Since, however, the social sciences each investigates a part of a single whole, their separation especially in general broad subjects that relate to questions of policy is extremely artificial and harmful. Such separation can only be tolerated in limited fields, so that considering a factor as a datum or as a variable should depend on two elements : a temporal element, the length of the period under consideration; and a spatial factor, the breadth of the subject under consideration.

35. *The assumption of perfect foresight* — The traditional way to evade the problem of expectations is to concentrate on the hypothesis of the stationary state. This is the method of the Austrians in general. On the assumption of stationary conditions, "we can reasonably assume that experience of these constant conditions will lead entrepreneurs to expect their continuance; so that it is not necessary to distinguish between price-expectations and current prices, for they are all the same". Thus, as a result of the assumption

1. Samuelson, *ibid.* p. 339.

of stationary conditions, we obtain a double gain in simplification: on the one hand, as a result of the persistence of the same conditions, expectations finally become through trial and error identical with the expectation of the position of equilibrium, i. e. become correct expectations; on the other hand, as a result of that same persistence, people have no basis for expecting future prices to change. We thus avoid all possibility of instability arising from the side of expectations. The assumption of perfect foresight logically follows from the assumption of a stationary economy.

36. *Static expectations* — A less drastic assumption is that of static expectations. This assumption states only that present prices are expected to continue, without necessarily implying that such expectation is correct; in other words, without assuming stationary conditions. The advantage of such an assumption is that it simplifies the problem of intertemporal relations of supply and of demand as we shall soon indicate.

The implication that expectations are not necessarily correct leaves room, however, for disequilibrium arising from the disappointment of expectations. In the Marshallian analysis, such disequilibrium persists until the position of equilibrium is reached. Since the temporary equilibrium, however, is not an equilibrium with reference to a longer period, equilibrium in the long run is the only position where the realisation of expectations is a persistent phenomenon (on the assumption that no disturbing outside influence takes place). This, however, is merely equivalent to the question-begging assumption of the stationary state, as was noted above.

37. *Expectations and Marshallian analysis* — The role of expectations in the Marshallian analysis is clearly explained by Lerner.[1] Both the supply and the demand schedules in the Marshallian analysis are groups of anticipated (ex-ante) positions with only the point of intersection as the consistent equilibrium point (ex-post) which is realised. This should be understood to mean that, at each 'false' price, the buyers and sellers plan the buying or selling of the quantities indicated by the demand and supply curves, but these plans are not realised until they coincide with the position

1. A. P. Lerner, "Ex-ante analysis and wage theory," *Economica*, vol. IV, new series, pp. 436—449.

indicated by the equilbrium point. This was, in fact, acceded by one of the proponents of expectation analysis, Ohlin, who finally characterises the Marshallian curves as a form of ex-ante analysis. The neglect of expectations in static equilibrium analysis should not, however, be underestimated. For as we have seen, the assumption of stationariness is ultimately nothing but an evasion. Similarly, the assumption of unitary elasticity of expectations has been a major source of classical neglect of the possibility of inherent instability in the economic system, as we shall see.

38. *Is expectation a dynamic element?* — In discussing above the place of expectations in economics, we pointed to that group of economists who refuse to incorporate its analysis in their systems. Of this group, Haberler calls expectations 'non-operational concepts' since their verification requires recourse to the method of introspection. Under the influence of modern behavioristic psychology, he points to the possibility of completely dropping this 'psychological link' between the present and the future in favor of explanations based on 'observable phenomena'. The only credit he gives to the concept is that it helps to remind us of the inaccuracy of the so-called laws of dynamics.[1]

Although it is disputable whether explanations based exclusively on a behavioristic approach are entitely satisfactory, we shall soon find that more recent tendencies in the field of economic dynamics are in the direction of eliminating all reference to the problem of expectations. The main objective of such a tendency is probably to make feasible the task of statistical verification. The problem of evaluating statistically the important dynamic relationships was handled in a number of pioneer studies by Tinbrgen with the objective of facilitating the construction of concrete dynamical models. The recent tendency in dynamic analysis, as is shown by Samuelson, seems to be in the same direction.

This, however, does not mean that the concept of expectations as an analytical tool is useless. On the contrary, the first step in progressing with dynamic analysis was probably by the more profound analysis of this phenomenon. Many writers consider the

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1. G. Haberler, *Prosperity and Depression*, 1941, pp. 252n, 254.

incorporation of expectation analysis into economic theory as "the stepping-stone to dynamic analysis".[1] The contribution of the so-called Swedish ex-ante analysis to the understanding of economic phenomena and to the solution of some of the most sterile controversies (e. g. the controversy around the equality of saving and investment) is only too well recognised. This, however, does not change the fact that many of the analyses that utilise the factor of expectations as a basic part of their system are no more than refinements of the simpler 'comparative statics' type of analysis.

39. *Business cycle theory and expectation economics* — The revolt against the inadequate treatment of expectations came about in the from of business cycle theories that tend to stress the influence of the psychological element. The distinction between real causes and psychological causes of the business cycle is rather one of emphasis, since every economic fact has its psychological aspect. Moreover, real and psychological causes are intimately related.

The stress of the role of expectations has become fasionable through the influence of the Swedish School and the Keynsian School; although fairness to the older writers demands the interpretation of their theories as referring to expected quantities even if they frequently failed to emphasise expectations.[2]

The earlier forms of psychological theories, however, tended to discuss only one aspect of the expectation problem, namely, the variation during the cyclical phases of the uncertainty of expectations. These are the theories of 'errors of optimism and pessimism'.

These errors are unlikely to play an important role where stationary conditions or perfect foresight are assumed. Yet, since this is not the case such errors do inevitably occur. The source of such errors is primarily attributable to errors in forecasting arising from the fact that production takes time and to the lack of coordination of entrepreneurial plans. The reason why such errors do not tend to cancell out, but instead tend to work in the same direction, is the common mutual generation of psychological attitudes; in other words, the fact that they are not statistically independent.

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1. Lundberg, *ibid*, p. 175.
2. Haberler, *ibid.*, p. 38n.

V — ALTERNATIVE WAYS OF BUILDING A DYNAMIC SYSTEM

40. *Dissatisfaction with statics* — By now, dissatisfaction with equilibrium economics must have become evident. It seems that equilibrium economics has fallen into the error of generalising from the fact that the actions of individuals (households or firms) tend towards some equilibrium norm, to the belief that the whole economic system tends in that direction. This dissatisfaction with equilibrium analysis has led many economists to propose different methods of attacking the problem of dynamics.

41. *Shall we build on statics?* — Once it is decided that some more adequate dynamical system of explanation has to be constructed, a fundamental question arises as to whether it is advisable to start with the basic static structure and introduce into it the necessary dynamic elements; or to adopt a more fundamental approach by replacing static assumptions by dynamic ones.

J. M. Clark decides to answer this question by reference to a criterion : If dynamic assumptions differ only in a mechanical (i. e. quantitative) way from the static ones, then they can be used and supplemented. He, however, tries to prove that they differ qualitatively (or chemically, as J. S. Mill calls it). This he does by surveying the assumptions proper to dynamics, concerning human motives, freedom of exchange and other institutions, the existence of 'collective' persons, the ethical forces, and so forth. These are found to be qualitatively different from the assumptions made by static analysis. His conclusion is that "dynamic study must not be cast in static molds".[1] This, however, does not mean that he considers statics as useless, for according to him statics still has its own place.

The same conclusion is arrived at by Kuznets in his above mentioned article. In fact, as he states it, one of the main objectives of that article was to advocate a more promising direction than that indicated by equilibrium theory. After having discussed the concept of time-differences, the basis for his assault on equilibrium analysis, he characterises that analysis as a "blind alley" and advocates the discard of the concept of equilibrium and simultaniety.[2]

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1. J. M. Clark, *ibid.*, pp. 47—8, 69—70.

2. S. Kuznets, *ibid.*, pp. 399, 414.

On the other hand, some writers argue that, since the attempt to find a new equilibrium is the motive behind dynamic reactions, the concept of equilibrium is an adequate basis for understanding empirical interrelationships (in the from of a moving general equilibrium). One of these writers, R. W. Souter, blames human limitations arising from the complexity of economic phenomena. The real problem of dynamics, he maintains, is, first, to make the analysis comprehensive, to include the whole economic system; and second, to make it quantitative, by finding the quantitative interrelationships of fundamental categories over time. We shall presently see that this position has many influential adherents.

A. The Economic process as a series of moving temporary equilibria

42. *The moving equilibrium* — From our previous discussion of the Marshallian method of analysis it must have become evident that Marshall's conception of the working of the economic system is one in which the system is always tending towards some equilibrium defined by the relevant initial conditions. The system, if left to its own endogenous tendency, is conceived of by Marshall as finally settling to a long run equilibrium. But the system is not left to its own endogenous influences, it is supposed to be continuously receiving shocks from factors outside that system; always more or less seeking after an equilibrium that in itself refuses to remain put. The concept is one of a moving equilibrium, but the movement of the equilibrium is attributed to factors outside the system.

A progressive step is made when the same concept of a temporary tendency towards an equilibrium which is constantly shifting is based on factors inherent in the system. Instead of escaping throught the easy device of 'exogenous' factors, factors from within are called upon for an explanation of dynamic movement. This step is achieved by expectation analysis such as that provided by Hicks.

In *Value and Capital,* Hicks asks the question: Is it possible in dynamics to use the same method of analysis as in statics? And he answers in the affirmative: "There is a way," he says,

"of reducing the dynamic problem into terms where it becomes formally identical with that of statics. This way is the treatment of the dynamic process as a series of moving equilibria".

To preserve the essentials of static analysis, Hicks adopts Marshall's method with some amendement. He discards the Marshallian tripartite division of adjustment time as suitable only for partial analysis, not for a general analysis involving the whole system. His analytical time-period is the 'week', defined as "that period of time during which variation in prices can be neglected." This operational week in not necessarily identical with the calender week.

The concept of the week makes possible the treatment of the process of change as a series of moving equilibria.

The influence of the future on present actions is taken account of through the concept of planning combined with the concept of expectations. According to Hicks' definition of the week, the week should be the planning period. He thus assumes that planning is made every week on a certain day which he calls Monday, the market day. It is admitted that this is not the way things happen in actual life, since, first, the planning period differs as among the different firms; and, second, since the willingness of the different planners to make major alterations in their plans also differs.

Taking account of the future, however, involves some difficulty arising from the fact that expected future magnitudes are uncertain, a fact which destroys the basis of comparability between these magnitudes and present magnitudes. To get around this difficulty, the most probable expected price is adjusted, positively or negatively, by an allowance for risk, which depends on the probability of the expected price, on the one hand, and on the dispersion of the probability distribution, on the other. Hicks admits, however, that such treatment is not entirely satisfactory. The two factors which are deemed not to promise to permit of treatment with his tools are, in the first place, the influence of the willingness to bear risk; and, in the second place, the influence of the riskiness of one part of the plan on the rest of that plan. We shall discuss below the influence of the element of risk on the attitude towards flexibility of plans.

43. *Planning at a point in time* — Planning at a point in time involves two steps : First, it is necessary to make a prognosis of the future, i. e. a calculation at the given point in time of the possible effects of actions that are intended to be undertaken in the future, noticing that an intended action at any point in time may impose a constraint on the freedom of choice among different patterns of behavior in the future. Second, once this prognosis of the future is made, the planner comes to the choice among different alternative plans, a choice which partly depends on his valuation attitudes. This latter problem of choice among alternative plans is the easier to dispose of, so we shall discuss it first.

44. *The choice among alternative plans : the assumption of maximising behavior* — Once the prognosis of the future has been made, and future magnitudes have been reduced to a comparable basis with present magnitudes, the problem of choice among alternative plans becomes easy to solve on the assumption of maximising behavior. The familiar outcome of such maximising behavior under static conditions is the rule that the planner equalise marginal cost and marginal revenue. Some writers apply the same rule under dynamic conditions (e. g. Harrod in the *Trade Cycle*). This is equivalent to assuming a hand-to-mouth behavior on the part of the planner. If the possibility of holding inventories is taken into account, the possibility of a time allocation of costs different from the time allocation of sales (neglecting the period of production) becomes evident.

The problem resolves itself into investigating the condition for maximising the present value, or for that matter the value at any other point in time (the maximum at one point remaining always a maximum since all compared magnitudes are proportionally discounted or accumulated towards the other points) of the difference between prospective outlays and prospective yields. The solution for such a problem is through the distribution of production among the various period of the plan so as to equalise discounted marginal costs and the distribution of sales so as to equalise discounted marginal revenues.[1] Maximum present value is attained by the equalisation of discounted marginal costs and discounted

1. A. Smithies, "The maximisation of profits over time with changing cost and demand functions," *Econometrica*, vol. 7, pp. 312—318.

marginal revenues. This rule is obviously logical since behavior according to the marginal principle, which is the meaning of rationality (Viner) requires, not only the equalisation at the margin of costs and revenues intratemporally, but also that equalisation intertemporally, the discount factor being allowed for, so that any reallocation through time of cost incurrence or receipts is unprofitable.

45. *The prognosis of the future: uncertainty of expectation.* The solution to the problem discussed above, depended, it should be noticed, on the assumption that some method has been devised to reduce uncertain expected magnitudes to a comparable basis with present magnitudes. This was arrived at in Smithies' article through the assumption of the certainty of prediction. Most writers, however, try to find better solutions.

In his *General Theory*, Keynes suggests a method of approach to the problem as follows. "By his expectation of proceeds (i.e. the entrepreneur's) I mean therefore that expectation of proceeds which, if it were held with certainty, would lead to the same behavior as does the bundle of vague and more various possibilities which actually makes up his state of expectation when he reaches his decision." [1] This is, however, too vague to be of any utility in analysing the problem of uncertainty.

The commonly accepted analysis of uncertainty is a little more sophisticated. Expected prices, it is acknowledged, are not 'subjectively certain', but are a set of possible values. The degree of uncertainty of the most probable price depends on the range (meaning by this the 'practical range', i.e. the range excluding the tails) of the probability distribution. The difference between this most probable price actually expected with uncertainty and the same price expected with certainty is called the 'risk premium'. To reduce an uncertain expected price to a certain one, the risk premium is added (for a seller) or deducted (for a buyer).

Such a method of reducing uncertain to certain prices is opposed by many writers. In the first place, we should bear in mind, in case we might not accept the idea of the measurability of psychological attitudes, that the concept of the probability distribution

1. J. M. Keynes, *The General Theory*, p. 24n.

of expected prices does not imply such measurability: the order of the probability can be taken as a mere ranking of the subjective acceptability of these expected prices to the planner. The denial of any possibility of finding the 'risk premium' (e.g. as in Hart) can be answered by the fact that dealings in the futures market are common. In fact, the psychological tinge which so many writers do not like about the treatment of uncertainty can be avoided in precisely the same way as in the theory of consumer demand, namely, through indifference analysis. An indifference map can be constructed for buyers or for sellers; the curvature of the indifference curves on such amap indicates the 'degree of the unwillingness to bear risk'.

The 'effective price', so-called, is thus "the most probable price discounted for risk", and is the relevant price for the choice among intertemporal alternatives.

The existence of uncertainty is of great importance since it is this factor which limits the period of planning of purchases and sales. This is because as planning extends into the future the degree of uncertainty grows at an ever increasing pace, while at the same time the planner becomes more and more unwilling to bear risk. This is the factor that limits the planner's 'economic horizon' to a finite period.[1]

46. *Flexibility of plans as a reaction to risk* — The discount of expected prices for risk is one way of reacting to the presence of uncertainty. The planner does not decide to carry out his plan unless, beside being optimum, it also covers the possible loss arising from the disappointment of his expectations. It is, however, absurd of a planner who is almost sure that he will have to make some changes in his plan as time goes on to make an unalterable plan depending on the coverage of possible loss from complete scrapping of his plan in case his expectations are disappointed. The cost of providing for a flexible plan should almost unexceptionally be less than that arising from such absurd behavior. This is why the reaction of planners to uncertainty is two-sided: On the one hand, they try as far as is feasible to make plans which will be capable of being adapted to changed conditions without

1. Lange, *Price Flexibility and Employment*, ch. 6.

much loss. On the other hand, in so far as this type of reaction does not completely offset the threat of loss, they require in calculating their alternative plans that this threat be covered. This reasoning should provide a sort of limitation on the concept of discounting for risk, since all risk is not covered by this expensive method. Only risk which could not be otherwise met is covered in this fashion. It is evident that both methods involve a cost over and above the cost that would be necessary under conditions of perfect certainty; but it is a necessary, socially justifiable type of cost, except probably in so far as such risk is created by social action.

47. *The movement of equilibrium: inconsistency of expectations*- Up to this point, the similarity between traditional static analysis and this type of analysis, aside from refinements, is obvious. The real progress in economic theory does not lie in analysing and refining the theory of planning at a point in time; it lies in the investigation of the development that follows such planning once it has been made. The great deficiency of traditional theory is that it slighted the complications of the process that follows by the assumption of stationary conditions, or, what is equivalent, at least in the long-run.

The theory of the moving equilibrium, on the other hand, treats the dynamic process as a series of moving equilibria, the source of the movement of which is the continuous disappointment of expectations arising from different causes of inconsistency. As we have seen, analysis in terms of the disappointment of expectations may be only a formal garb which hides beneath it the implication of equilibrium analysis if the process of disappointment is conceived of as leading to a final long run equilibrium. This, however, is not what underlies the theory of moving equilibrium. The process of change in real factors is continuous; and along with it, the process of the disappointment of expectations.

The source of the inconsistency of expectations may lie in the inconsistency of price expectations, the inconsistency of plans, or the incorrect foresight of wants. An exaggerated sense of risk i. e. a lack of confidence, may be another source of imperfection. Out of these possible sources of disequilibrium, the first two, however, tend to be offset, though never completely, through the coordinating influence of forward trading. The need for the speculator on the

forward market is explained by Hicks as arising from the less persistent need to hedge planned purchases than planned sales, since new processes can usually be postponed"[1]. This alledged tendency of speculation to stabilise the economic system is severely contested by many writers. We have already referred above to the possibility of its being more of a destabiliser. It might however be more conducive to an understanding of the problems of speculation to distinguish between the ideal in itself and its possible misuses in practice.

In analysing the effect of current development on expectations a useful tool has been devised by Hicks, namely, the concept of the elasticity of expectations. This is defined as "the ratio of the proportional rise in the expected future price of x to the proportional rise in the current price." Rise is to be understood as either positive or negative. The use of this tool will become clear when we come to discuss the question of the stability of the system in development.

48. *Estimate of the method of moving equilibrium* — In a review of Hicks' book, Hawtrey condemns his method as no more than a glorified statics. "In parts III and IV", says Hawtrey, "prof. Hicks turns to 'Dynamic Economics'. His method, however, amounts to little more than a glorified statics. He posits a series of equilibrium positions at short intervals (called 'weeks' though not necessarily actual weeks), and the admixture of dynamics consists in the inclusion of forecasts of prices and of rates of interest in the motives determining equilibrium. The essentials of a dynamic system, the investigation of a state of disequilibrium, and the relative rates of progress of the corrective tendencies set up, he hardly touches on."[2]

This criticism on the part of Hawtrey is unfair. Hicks recognises that over time the system is in continuous disequilibrium. So much is distinct improvement on theories which conceive of the economic system as tending to a long run equilibrium. If this is not deemed sufficient, the only remaining possiblity is to conceive

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1. J. R. Hicks, *op. cit.*, pp. 135—9.

2. R. G. Hawtrey, *Journal of the Royal Statistical Society*, vol. 102, pp. 308—9.

of the economic system as being in disequilibrium even with reference to the short period (e.g. Hicks' week). In other words, attempts to adjust to successive positions of disequilibrium should be conceived of as continuous. Besides being unrealistic, such continuity is impossible. Even our consciousness of the element of time itself is discontinuous. However small the period chosen for which no adjustment takes place, it still is an equilibrium period, and perfect continuity remains an illusion. On the other hand, human capacity for the registration of events, and even more so, for the evaluation of these events and the amendement of plans accordingly is far from being perfect. Thus the choice of the unit period for the analysis as a period of registration, a period of planning, or any other period, is a matter of method and not of principle, and Hawtrey's criticism appears to be not well-conceived.

However, Hawtrey is right in criticising Hicks for paying little attention to the problem of time-differences. The treatment of these has been best undertaken by 'sequence analysis'.

B. Sequence Analysis

49. *Time differences* — We have seen that equilibrium analysis takes account of time differences by examining equilibrium after different spans of time, through the division of adjustment time into short and long periods. It is assumed, however, that in the short period, supply and demand are simultaneous variables. In fact, they vary consecutively; the sequence of variation is of considerable importance. We have already discussed one method of attacking the problem of time-differences, namely, cobweb analysis. In the present analysis, however, the problem is given fuller treatment.

50. *Aspects of time differences* — Time differences have two aspects. The first is the existence of lags, i. e. of retarded reaction to certain occurences; the second is the prevalence of disproportionality of effects either because these effects, although equal in absolute terms, occupy different time spans, so that considered as rates per time unit they are disproportional; or, because in absolute terms, although they may occupy the same time-spans, they are disproprortional. The latter type of disproportionality is the result of

the cumulative effects of certain stimuli, which give birth to continued lack of equilibrium[1].

The problem is thus one of the timing of reactions and their distribution over time. To account for both these elements, reaction must be referred to time periods. This is not merely a formal problem, but a real issue as we shall soon see.

51. *The unit period* — The choice of the unit period is the first step in sequence analysis. This choice is of analytical importance since both too short and too long a period are undesirable. A very short period is undesirable because reactions do not take place to very small changes. This is the phenomenon we referred to above when discussing the problem of discontinuity of reactions. Moreover, a very small period would amount to a partial analysis since then other things remain almost unchanged. The analysis loses all explanatory value as to causality and becomes a mere registration of events.

On the other hand, a very long unit period will not take account of important changes since changes that happen within the period are neglected. The concept of the unit period is thus relative : it is a sort of operational time.

The concept of a unit period, together with the assumption as to the pattern of quantitative variability of reactions with respect to time, take care of the distribution of these reactions over time. Thus there remains one more problem: the problem of time-lags.

52. *Time-lags* — Among the infinite number of time-lags, certain possibilities are picked out as being fundamental. Then, the plausibility of the sequence depends on the choice of such time-coefficients as well as on the other simplifying assumptions. The fundamental time-lags are the following: The first is the production lag, i. e. the lag between the incurrence of costs and the resulting output. The second is the expenditure lag, i. e. the lag between the paying out of costs and their expenditure by the income recepients. The third is the income lag, i. e. the lag between the expenditure of income and its reappearance as income through the productive process.

On the basis of the choice of the time lag, different model sequences have been constructed. We shall try to illustrate this by sample model sequences.

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1. S. Kuznets, *op. cit.*

53. *Model sequences based on the production lag* — Wicksell's cumulative process based on the discrepancy between the market rate of interest and the real rate of interest, is a pioneer work in sequence analysis. He starts with a stationary state. Entrepreneurs borrow from the banks the existing quantity of money to finance current production, only to receive the same amount back (plus interest, if interest is admitted to exist in the stationary state, which is denied by some economists like Schumpeter) in the form of sale-receipts to pay back their debts to the banks. Here Wicksell notes that the existence of the production lag may be neglected : production and expenditure may be considered simultaneous.

In the dynamic state, however, the importance of the production lag reappears. When, as a result of a discrepancy between the market and the natural rates of interest, say, as a result of lowering the former below the latter, the entrepreneurs start borrowing for expansion, they pay out to the factors of production additional income for which no equivalent in goods is coming forth on the market, at least for a while. Prices rise, with wages following on their heels, in an infinite upward spiral. The production lag has started the whole process.

The same idea underlies Keynes' model sequence in his pure theory of the credit cycle in the *Treatise on Money*. In order to finance an assumed expansion in consumer goods production, banks have to create credit since the saving-investment sphere is assumed to be in equilibrium. The created credit is injected into the income stream with no simultaneous increase in production, and the price spiral starts. Only when production starts to come forth do prices tend to go back to their previous level.

The production lag has also been utilised by E. Lundberg in his model sequences in the *Theory of Economic Expansion*. Lundberg considers the consumer's lag as unimportant, in the sense that he chooses to assume that people dispose of their incomes in the same unit period in which they receive them. In other words, the consumption lag is assumed to be always within the period. His unit-period is the production planning period. At the beginning of each period, producers plan their outputs on the basis of the experience of the preceding period, their output being both for covering expected sales and for keeping up their inventories at a normal

level. On the basis of these assumptions, Lundberg shows the type of fluctuations that are liable to arise from trying to maintain production at the assumed level. In this model, the disappointment to the planned production appears in the form of unintended changes in investment. We shall soon see that this is not the only alternative nor in fact the true picture of real life. In all this analysis, Lundberg uses numerical examples to illustrate his models, with different assumed quantitative relationships.

54. *Model sequences based on the expenditure lag : The Robertson sequence* — In contrast to the Lundberg model, Robertson bases his model on the importance of the expenditure lag, the other two lags, the production lag and the income lag, are considered to be unimportant.

Robertson divides time into short periods which he calls days. He then proceeds on the following assumptions : First, that the consumption of one period is determined by the income of the preceding period. Second, that the level of output and hence of income payments in one period depends on the sales of the same period. The discrepancy between intended savings and investment is here assumed, in contrast to the Lundberg model, to be solely in the form of unintended changes in savings.

Roberson applies the above concept of the importance of the expenditure lag in his analysis of the conditions for an even expansion. On the assumption that people keep a certain proportion of their income in the form of cash balances which they do not spend in the same unit period in which they received that income, Robertson goes on to prove that financing an assumed expansion with created credit will not be inflationary so long as the created credit does not exceed the "induced lacking" of circulating money which arises from the rise in income.

55. *A model sequence based on the income-payment lag* — The Swedish economist Johannson has built a model sequence based on the choice of the lag between expenditure and the response of production (and hence income-generation) as the fundamental lag. Although his starting point, the assumption of general rise in the wage level which gives birth to an expansion on the last stage of production (the nearest to the consumer) is questionable, as Lund-

berg pointed out, since the profit in the last stage is certain to be temporary, yet an expansion of consumer demand can be assumed on one basis or another, as for example on the basis of an expansion of consumer credit facilities. This expansion of consumer demand raises the prices of consumer goods in the last stage of production, a rise which is transmitted to the more remote stages (from the consumer) as a result of the increased demand from the previous stages, until the price rise reaches the first stage of production, when the effect of the rise will be an increase in output. A wave in the reverse direction (i.e. heading to the final stage) starts in which each stage starts to expand production, financing its expansion with created credit, prices rising (as a result of the production lag) until they finally fall when the last stage expands its production of consumers goods. The first part of the model is a clear illustration of a model based on the income generation lag.

26. *Evaluation of the method of sequence analysis* — From the above, it is to be noted that sequence analysis and the moving equilibrium analysis are fundamentally similar in that they both rely on a unit period within which variation is supposed to be negligible. The advantage of the former analysis over the latter is that the existence of time lags is incorporated as a main feature. We must not forget, however, that the source of the dynamic movement of the system lies primarily in the concept of the disappointment of expectations. In the following discussion, we shall see a new type of analysis in which this whole approach is discarded for a new type in which the source of instability is simply and solely the patterm of quantitative variation through time which is assumed.

C. Recent developments in dynamic analysis

27. *Stability analysis and dynamics* — During the present decade, outstanding progress in the analysis of economic dynamics was achieved through Samuelson's contribution to this field. The track that led into the formulation of the new theory of dynamics was the attempt to investigate the conditions for the stability of the economic system. As we shall soon see the first step in stability analysis was taken by Walras. Hicks then generalised the Walrasian stability conditions to the case of general equilibrium. And, finally,

Samuelson took the third step by making explicit the implication underlying the Hicksian and Walrasian analyses. Before we go into the detail of the analysis we want first to decide what we mean by stability.

58. *The meaning of stability* — The concept of stability is by no means a uniquely defined concept, although in economics it has come to take a conventional meaning among many writers. Frequently, a dynamic system is considered stable so long as its fluctuations remain bounded (i. e. do not go out to infinity). The term may be used also to mean approximately repetitive movement of a dynamic system. The conventional meaning of stability, however, is that every motion of the dynamic system approaches the position of equilibrium in the limit.[1] In this sense, Hicks defines stability as that quality in the dynamic system which makes "a slight movement away from the equilibrium position ... set up forces tending to restore equilibrium".[2]

This, however, is what is meant by stability in a general sense. Samuelson calls this type of stability "perfect stability of the first kind". It is called of the first kind to distinguish it from stability of the second kind which is stability in the sense of bounded fluctuation as indicated above. Both of these can be either in the small or in the large, the first meaning stability only with respect to small displacements from equilibrium.[3] In addition to this classification of the different types of stability, we shall later come to the discussion of stability of a certain order or of a certain rank, a distinction adopted by Lange [4].

59. *The Walrasian stability conditions* — According to Walras, it is necessary and sufficient for an equilibrium position to be stable that, in the neighbourhood of that equilibrium position, "a price above the equilibrium price causes excess supply, and a price below it causes excess demand." In other words, the Walrasian stability condition is that the excess demand function be negatively

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1. See Samuelson, *op. cit.*, pp. 333—4 and p. 261.
2. *Value and Capital*, p. 62.
3. *Foundations*, pp. 261—2.
4. *Price Flexibility and Employment*, Appendix.

sloping. From this starting point, a dynamic model can be constructed to show the pattern of variation of price with respect to time, and by mathematical manipulation this condition can be found to imply that the slope of the supply curve must be greater algebraically than that of the demand curve as a condition for stability. [1]

Samuelson shows that this is not the true dynamic stability condition, in the sense that it is not general enough but applies only to the specific model chosen by Walras. He goes on to show that, with other models as a starting point (e. g. the Marshallian model implied in his theory of long run normal price or that underlying his offer curves in the *Pure Theory of Foreign Trade*, or the cobweb models), different stability conditions can be arrived at. In addition to this limitation, the Walrasian stability condition is fit only for partial analysis since it neglects the effect of the variation of one commodity's price on other prices.

60. *The Hicksian stability conditions* — Hicks stepped in to remedy this lack in the Walrasian condition by calling an equilibrium stable only if a price other than the equilibrium price results in the same Walrasian consequences "when all other prices have adjusted themselves so as to maintain equilibrium in their respective markets". According to Hicks, a system of multiple exchange is perfectly stable if a rise in the price of a commodity causes an excess-demand: First, when all other prices do not change; Secondly, when some of the other prices change; Thirdly, when all the other prices change so as to maintain equilibrium in their own markets. If the first or the second conditions are not realised (the third is indispensable), the equilibrium is called 'imperfectly stable'.[2]

Lange calls 'partial stability' what Hicks calls imperfect stability. According to Lange, partial stability 'of order m' designates stability when only m prices are adjusted (the rest of the prices remaining constant). He calls this partial stability 'of rank m' if, in addition to the above, higher orders than m are unstable. In this sense, Hicks' perfect stability is the extreme form of partial

1. See Samuelson, *op. cit.*, p. 263.

2. *Value and Capital*, p. 248.

stability where the order and rank are of the same degree as the number of commodities.[1]

On the basis of the Hicksian stability conditions, Lange introduces his concept of the positive monetary effect, which is the core of his analysis in *Price Flexiblity and Employment.* The monetary effect (positive or negative), is, in fact, the reverse side of Hicks' excess demand for a commodity (negative or positive), since, as Lange points out, an excess supply of a commodity is equivalent to an excess demand for cash balances, and vice versa. The Hicksian stability condition reduces to the form that equilibrium is stable only if the divergence of the price of the commodity (or factor) from the equilibrium price results in an excess demand for cash balances (negative or positive) when all other prices have changed proportionally, a result which tends to work in the reverse direction to that of the original change in price.

We shall not discuss in detail the factors which help to stabilise the economic system by helping to bring about a positive monetary effect. It suffices to enumerate here these factors, which are, first, the inelasticity of (expectations except probably for the case of the interest rate); second, the existence of free international trade; and third, the absence of oligopolistic and oligopsonistic behavior.[2]

61 *Samuelson's dynamic stability conditions* — In order to judge the question of the sufficiency and necessity of the Hicksian stability conditions, Samuelson develops a system of dynamics of his own. This system is based on making explicit the assumption implied both in the Hicksian and in the Walrasian analyses that an excess demand for a commodity makes its price rise and an excess supply makes its price fall. In the Hicksian and the Walrasian analyses this obviously is assumed since the excess demand or suply required by their stability conditions are not important in themselves but because they help to restore the price to its equilibrium position. This process of the restoration of the price to its equilibrium position is precisely the core of Samuelson's analysis of dynamic stability. His assumption with regard to that process is quite simple: He makes the rate of change in price in relation to

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1. Lange, *op. cit.*, p. 93.

2. For details, see Lange, *op. cit.*

time a function of the excess demand or supply of the commodity. With this step taken, everything else in his system follows. Lange shows in a clear sequence how Samuelson's dynamic stability conditions are derived from this basic assumption.[1]

62. *Linear and non-linear systems* — Most economists in the past have been concerned with linear systems, with the exception of cobweb analysis. Linear system analysis is probably justified when the analysis is primarily concerned with stability in the small (i.e. for small displacements); and although the system is actually and in fact not linear. Analysis of stability in the small is certainly relevant since first order stability does depend on stability in the small.[2]

However, the analysis of non-linear systems, in spite of their complication, offers great advantages. Non-linear systems explain for the first time how fluctuations can be of a fixed amplitude regardless of the initial displacement. Moreover, non-linear systems, especially non-linear difference equations, offer such a striking variety of possible instability patterns as could never be thought of under linear systems.[3]

VI — SUMMARY AND CONCLUSION

63. In the preceding survey of the various methods of treating the time element in economic analysis, we have shown the various stages through which efforts in this field have passed. The development of analytical technique in this respect shows a definite progressive trend that cannot be doubted. This should not deter us, however, from making two critical remarks concerning this development.

The first relates to the role of expectations in modern dynamic analysis. Recent developments in dynamic analysis have the appearance of eliminating the role of expectations as an influence deter-

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1. Lange, *op. cit.*, Appendix, pp. 94—6.

2. See Samuelson, p. 290.

3. See Samuelson's diagram on p. 303 and his table on p. 305. This qualitative behavior which he illustrates there is derived from the 'simple' difference equation on p. 303.

mining the course of the economic system. This, however, is not so in fact. For inasmuch as expectations are based on past experience, lagged values of the variables can be incorporated in the dynamic system as an expression of expectations. The only feature in current dynamic analysis for which it can be reproached in this regard is its tendency to confine such lagged values to those of the variable the expected values of which are in question. This is undoubtedly too simple a view of expectations and of the way in which they are formed. Lagged values of other variables which have bearing on the expected future values of a particular variable should be included in the explanatory system.

The second remark relates to the empirical import of current dynamic analysis. Here it should be noted that this analysis offers a rich variety of possible dynamic models without indicating which among this large variety of possibilities are the relevant models for the explanation of the behavior of actual economic systems or of particular portions of the time path of paticular economic systems. This glaring flaw in the efforts of current dynamic analysis was only too apparent in experience with respect to economic forecasting. This only means that for dynamic analysis a formidable task still lies ahead, namely, that of the econometric 'verification' of the various economic models which that analysis suggests. Unless and until such verification is undertaken, efforts in dynamic analysis will not have gone past the stage of mental exercises of little practical significance, along the same pattern as traditional pure theorizing in the economic field.